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A model for life prediction in low-cycle fatigue with hold time

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In high-temperature and low-cycle fatigue, creep damage reduces fatigue life. In this investigation, a model for life prediction in low-cycle fatigue with hold time at tensile peak strain is suggested in the temperature region of $0.5T_m$. This model is based on previously reported theories for creep cavitatation and we predict the creep–fatigue life. It is proposed that the fatigue life may be predicted in terms of plastic strain range, test temperature, hold time and other parameters. An equation for life prediction is given and checked using other investigators' experimental results with various hold times. The predicted creep–fatigue lives are in good agreement with those observed experimentally for 304 stainless steel, 316 stainless steel, CrMoV steel and 13CrMo44 steel.

1. Introduction

Low-cycle fatigue behaviour at high temperature is of considerable interest in the design of many components used in power generation and aeronautics. It has become apparent that fatigue lives depend not only on testing temperature but also on wave shape. In studies on creep–fatigue interaction, it is important to quantify and to predict fatigue lives. Recently, some methods for predicting fatigue life have been proposed.

Under low-cycle fatigue conditions, the relation between the number of cycles to failure (N_f) and the plastic strain range ($\Delta\varepsilon_p$) is known to be $(N_f)^\alpha \Delta\varepsilon_p$ = constant, which is the Coffin–Manson relationship [1, 2].

Rie [3] has shown that, instead of using N_f in the Coffin–Manson relationship, the critical number of cycles to failure, N_{cr} (the number of cycles leading to the formation of fatigue damage that noticeably weakens the specimen), can be used for better fitting of the Coffin–Manson relationship.

When strain or stress is held for a time at the tensile peak of loading cycles, degradation of fatigue endurance is frequently observed in stainless steel, Cr–Mo steels and superalloys [4, 5].

In ASME code case 1592 "the linear damage summation rule" has been suggested for life prediction on the basis of the assumption that fatigue and creep do not interact with each other: if the linear sum of fatigue damage plus creep damage reaches a critical value, then the material fails. The linear summation rule has been applied to 304 and 316 stainless steels, $2\frac{1}{4}$Cr–Mo steel and Incoloy 800, but it proved to be inadequate for life prediction especially when creep and fatigue interact nonlinearly with each other [6–9].

Three phenomenological models have been proposed for prediction of fatigue life under combined creep–fatigue damage conditions: the first is the "frequency-modified Coffin–Manson law" [10, 11], the second is the "strain range partitioning method" [12, 13] and the third is the method of "damage functions based on hysteresis energy" [14, 15]. In addition to the above three phenomenological models, two more models have been suggested on the basis of micromechanistic considerations for life prediction under combined creep–fatigue damage conditions: one is that proposed by Tomkins and Wareing [16] and the other is the method of "interactive damage rate equations" [17–19]. In the practical application of these models, however, there are many difficulties in obtaining the various parameters and constants in the equations.

To have a more reliable method of life prediction, one needs to understand the micromechanistic damage formation mechanism under creep–fatigue interaction conditions.

For materials used under high-temperature fatigue conditions, especially with a hold time, it is well known that life is reduced very significantly by nucleation and growth of a cavity along grain boundaries and is accompanied by a change in the mode of failure from transgranular to intergranular [8, 20]. By performing fatigue life tests in an ultra-high vacuum of 10^{-8} torr, it was found that, above a certain transition temperature, creep–fatigue failure is controlled by general creep damage rather than by a process of fatigue crack initiation and propagation [21].

In developing a model for life prediction of a material under this type of creep–fatigue failure, one has to think about what type of cavities are formed and why they could be formed in a particular area. Generally, round-type cavities are found along grain boundaries at high temperature ($\sim 0.5T_m$) and for an unbalanced hold cycle, especially a tension hold cycle [22]. On the other hand, wedge-type cavities are found at intermediate temperature ($\sim 0.4T_m$) and for an unbalanced triangular wave shape, in particular a slow–fast cycle.

In this investigation, a model is derived for life prediction in terms of round-type cavitation at or above $0.5T_m$ during low-cycle fatigue with an unsymmetrical hold time by modifying the previously reported theories for creep cavitation. The model is compared with the experimentally obtained fatigue lives reported by other investigators.

2. A model for life prediction in low-cycle fatigue with hold time

When modelling for life prediction in low-cycle fatigue with hold time, it is necessary to estimate the accumulation of area of round-type cavities in each cycle. It is hypothesized on the basis of scanning electron microscope observations that round-type cavities are nucleated independently from each other. The nucleation of cavities in grain boundaries was analysed in terms of classical nucleation theory [23]. The analysis provided an explanation for why cavities form preferentially in grain boundaries at elevated temperature. It is known that, under tensile loading, cavities can be formed by vacancy clustering [24–26], grain boundary sliding [27, 28] and dislocation pile-up [29, 30]. However, even in a crept specimen with a very significant amount of grain boundary sliding, the rate of wedge-type cavitation is known to be slower than that of round-type cavitation by a factor of 0.1 [31]. Therefore, it is hypothesized that round-type cavities are formed by vacancy clustering, dislocation pile-up and/or grain boundary sliding.

It is known that there is continuous nucleation of cavities with plastic strain [32] and fatigue cycle [33]. According to these, it is assumed that cavities are formed in every cycle and that the number of cavities in one cycle is proportional to the plastic strain range. This is represented by

$$n = P\Delta\varepsilon_p N \quad (1)$$

where P, $\Delta\varepsilon_p$ and N are the nucleation factor per unit area of grain boundary, the plastic strain range and the number of cycles, respectively and n is the number of nucleated cavities during cyclic loading per unit area of grain boundary.

These generated cavities are assumed to grow mostly during the hold time period in tension straining by grain boundary diffusion of vacancies. The creep strain rate measured from load relaxation during tension strain hold is obtained experimentally to be of the order of 10^{-9}. In this slow creep deformation rate, it is known that cavity growth is controlled by vacancy diffusion along the grain boundary [34].

The original model of diffusional growth of cavities at the grain boundary [35] has recently been modified to take account of more realistic boundary conditions [36, 37]. However, these refined equations are difficult to handle analytically. Nevertheless, when the cavity spacing is large compared with the cavity radius,the Hull and Rimmer relation [35] provides a good approximation of cavity growth:

$$\frac{dA}{dt} = \frac{2\pi D_g \delta \Omega \sigma}{lkT} \quad (2)$$

where A is the area of a given cavity, l is the cavity spacing, D_g is the grain boundary diffusion coefficient, δ is the width of grain boundary for enhanced diffusion, Ω is the atomic volume, σ is the stress, k is Boltzmann's constant and T is the absolute temperature.

Load relaxation occurs during tensile strain hold to give creep strain. Therefore, the stress term in Equation 2 has to be modified to be a function of hold time, i.e.

$$\sigma = \sigma(t) \quad (3)$$

Equation 3 may be substituted into Equation 2 to give the growth rate of cavity area with time:

$$\frac{dA}{dt} = \frac{2\pi D_g \delta \Omega \sigma(t)}{kTl} \quad (4)$$

If dA/dN is the area increment per cycle for a given cavity, Equation 4 may now be integrated with time and dA/dN is given by

$$\frac{dA}{dN} = \frac{2\pi D_g \delta \Omega}{kTl} \int_0^t \sigma(t)\, dt \quad (5)$$

where t is a period of hold time.

In practice, cavities are believed to be nucleated continuously during the cycle. However, the experimental measurement of growth rate is complicated and, therefore, the growth rate for a given cavity is assumed to be dependent on total cavities per unit area in a given experimental condition. Thus in Equation 5, the average cavity spacing, l, may be considered to be proportional to $(1/n)^{1/2}$, where n is the number of cavities per unit area of grain boundary, so that Equation 5 can be rewritten as

$$\frac{dA}{dN} = \frac{2\pi D_g \delta \Omega (P\Delta\varepsilon_p N)^{1/2}}{kT} \int_0^t \sigma(t)\, dt \quad (6)$$

From Equation 6, the increment in the cavitated area, A_1, for a given cavity formed in the first cycle up to N cycles be given by

$$A_1 = \int_0^N P^{1/2} \Delta\varepsilon_p^{1/2} N^{1/2} \frac{2\pi D_g \delta \Omega}{kT} \left(\int_0^t \sigma(t)\, dt \right) dN$$

$$= \tfrac{2}{3} P^{1/2} \Delta\varepsilon_p^{1/2} N^{3/2} \frac{2\pi D_g \delta \Omega}{kT} \int_0^t \sigma(t)\, dt \quad (7)$$

It is assumed that cavities are formed in every cycle and that the number of cavities per unit area of grain boundary in one cycle is $P\Delta\varepsilon_p$. From this and Equation 7, the total cavitated area per unit area of grain boundary, A_t, up to N cycles will be given by

$$A_t = P\Delta\varepsilon_p (A_1 + A_2 + \ldots + A_N)$$

$$= P\Delta\varepsilon_p \left(\int_0^N G dN + \int_1^N G dN + \ldots + \int_{N-1}^N G dN \right)$$

where

$$G = P^{1/2} \Delta\varepsilon_p^{1/2} N^{1/2} \frac{2\pi D_g \delta \Omega}{kT} \int_0^t \sigma(t)\, dt$$

i.e.

$$A_t = \tfrac{2}{5} P^{3/2} \Delta\varepsilon_p^{3/2} N^{5/2} \frac{2\pi D_g \delta \Omega}{kT} \int_0^t \sigma(t)\, dt \quad (8)$$

Equation 8 indicates the total cavitated area (per unit grain boundary area) as a function of the various experimental parameters.

At the critical number of cycles to failure (N_{cr}), the load-carrying capacity is drastically reduced by coalescence of grain boundary cavities and/or unstable crack growth. Creep–fatigue failure is then controlled by creep damage rather than by a process of fatigue crack initiation and propagation [21]. Therefore, a failure criterion of this work is established by assuming that, in any condition, catastropic fracture occurs at N_{cr} when the total cavitated area reaches a critical value. Thus from Equation 8, the number of cycles to failure is given by

$$N = C(\Delta\varepsilon_p)^{-3/5} \left(\frac{\exp(-Q_g/RT) \int_0^t \sigma(t)\, dt}{T} \right)^{-2/5} \quad (9)$$

where T is the test temperature, Q_g is the activation energy of grain boundary diffusion, t is hold time and C is a constant including the critical cavitated area. The value of N in this equation is the critical number of cycles to failure and, if we know the value of C, we can predict the life of a specimen under creep–fatigue interaction.

3. Discussion

The proposed model is suitable for creep–fatigue life prediction if the failure of materials occurs by round-type cavitation damage. The proposed equation has been checked using experimental results obtained independently by other investigators [38–42]. There are numerous test results for creep–fatigue interaction. However, only the above-mentioned results could be used to check the model, simply because only they have data for stress relaxation during hold times. In the above-mentioned papers, there is no mention of the value of N_{cr}, but instead cycles to failure, N_f, only are reported. In practice, even though the load-carrying capacity of the specimen is drastically reduced after N_{cr}, however, N_{cr} is equal to or above $0.98N_f$ in a low-cycle fatigue with hold time. Therefore, we may use N_f instead of N_{cr} to check our model.

Ermi and Moteff [38] performed creep–fatigue tests with AISI 304 stainless steel. Their creep–fatigue data are shown in Table I. From experiments performed at a temperature of 866 K, plastic strain range 0.72% and tension hold time of 10 min, the result indicates that $C = 1.9 \times 10^{-1}$ by putting N_f, $\Delta\varepsilon_p$, T, t, Q_g and $\sigma(t)$ into Equation 8. From this calculated constant C, we predict the creep–fatigue life with another hold time and plastic strain range. The predicted creep–fatigue lives are compared with experimental lives and are given in Fig. 1. This plot shows that the predicted lives are in good agreement with the experimental lives.

As in the preceding analysis, some other experimental data were used to check the model. Tables II to V show the data and Figs. 2 to 5 show the predicted life plotted against the experimental life for 304 stainless steel [39], 316 stainless steel [40], CrMoV steel [41] and 13CrMo44 steel [42]. The agreement between predicted lives and experimental lives for the four materials is quite good. As shown in Fig. 6, a linear relation between fatigue life and $\int_0^t \sigma(t)\,dt$ normalized by temperature and plastic strain range was obtained in a log–log plot and the slope of this line is equal to -0.4 for all materials. The relation can be described by the following formula:

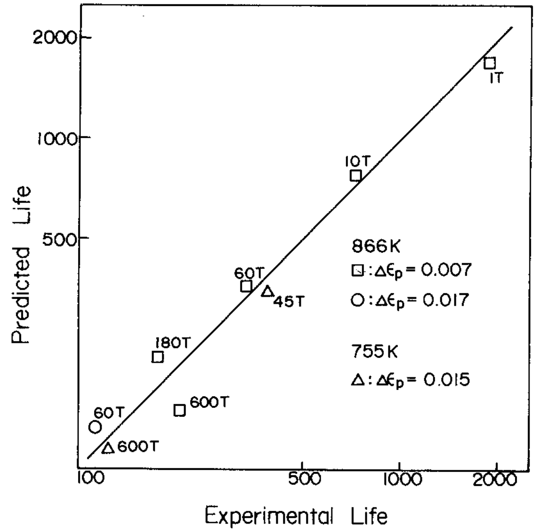

Figure 1 Predicted life against experimental life for 304 stainless steel [38].

$$N_f = C\left(\frac{\int\sigma(t)\,dt\exp(-Q_g/RT)\Delta\varepsilon_p^{3/2}}{T}\right)^{-0.4} \quad (10)$$

Equation 10 is identical to Equation 9. C is a material-dependent constant. This result indicates that fatigue failures occur by the cavitation damage represented in our model.

The excellent agreement between predicted and observed fatigue lives argues strongly that the model suggested for the role of creep

TABLE I Fatigue test data on 304 stainless steel [38]*

Temperature (K)	Hold time (min)[†]	Total strain range, $\Delta\varepsilon_t$	Plastic strain range, $\Delta\varepsilon_p$	Tensile peak stress (MPa)	Fatigue life, N_f	$\int_0^t\sigma(t)\,dt$ (MPa sec)	Predicted life, N_{cr}
866	1T	0.01	0.007	252	1748	1.49×10^4	1695
	10T	0.01	0.0072	231	706	1.28×10^5	706
	60T	0.0099	0.0072	235	338	7.56×10^5	347
	180T	0.0102	0.0076	224	170	2.11×10^6	233
	600T	0.0103	0.0082	217	212	6.97×10^6	132
	60T	0.0198	0.0166	322	112	1.08×10^6	165
755	45T	0.0202	0.0152	412	388	1.02×10^6	342
	600T	0.0198	0.0154	398	123	1.30×10^7	114

*$Q_g = 195\,\text{kJ mol}^{-1}$ [43].
[†]T = hold time only at tension.

TABLE II Fatigue test data on 304 stainless steel [39]*

Temperature (K)	Hold time (min)†	Total strain range, $\Delta\varepsilon_t$	Plastic strain range, $\Delta\varepsilon_p$	Tensile peak stress (MPa)	Fatigue life, N_f	$\int_0^t \sigma(t)\,dt$ (MPa sec)	Predicted life, N_{cr}
866	1T	0.01	0.0069	253.4	3034	1.49×10^4	3005
	5T	0.01	0.0072	240.0	2222	6.69×10^4	2222
	10T	0.0099	0.0072	226.8	1826	1.29×10^5	1655
	60T	0.01	0.0078	195.8	767	6.95×10^5	764

*$Q_g = 195\,kJ\,mol^{-1}$ [43].
†T = hold time only at tension.

TABLE III Fatigue test data on 316 stainless steel [40]*

Temperature (K)	Hold time (min)†	Total strain range, $\Delta\varepsilon_t$	Tensile peak stress (MPa)	Fatigue life, N_f	$\int_0^t \sigma(t)\,dt$ (MPa sec)	Predicted life, N_{cr}
873	10T	0.014	164.0	170	1.00×10^5	153
	30T	0.014	258.0	92	3.38×10^5	94
	100T	0.014	251.5	57	1.42×10^6	53
	500T	0.014	249.0	30	4.00×10^6	35
	30T	0.025	263.5	52	4.27×10^5	48
	100T	0.025	261.0	39	1.25×10^6	34
	500T	0.025	259.0	18	6.22×10^6	20

*$Q_g = 187\,kJ\,mol^{-1}$ [44].
†T = hold time only at tension.

TABLE IV Fatigue test date on CrMoV steel [41]*

Temperature (K)	Hold time (min)†	Total strain range, $\Delta\varepsilon_t$	Tensile peak stress (MPa)	Fatigue life, N_f	$\int_0^t \sigma(t)\,dt$ (MPa sec)	Predicted life, N_{cr}
838	10T	0.01	275.0	444	1.28×10^5	478
	30T	0.01	287.0	286	4.08×10^6	294
	100T	0.01	293.0	177	1.73×10^6	165
	180T	0.01	275.0	140	2.56×10^6	141

*$Q_g = 182.3\,kJ\,mol^{-1}$ [45].
†T = hold time only at tension.

TABLE V Fatigue test data on 13CrMo44 steel [42]*

Temperature (K)	Hold time (min)†	Total strain range, $\Delta\varepsilon_t$	Plastic strain range, $\Delta\varepsilon_p$	Tensile peak stress (MPa)	Fatigue life, N_{cr}	$\int_0^t \sigma(t)\,dt$ (MPa sec)	Predicted life, N_{cr}
873	1T	0.01	0.0057	332.0	1180	1.69×10^4	1354
	10T	0.01	0.0059	306.0	555	1.81×10^5	572
	30T	0.01	0.0061	301.5	402	4.94×10^5	387
	10T	0.02	0.016	324.0	265	1.79×10^5	297
	30T	0.02	0.0165	313.0	182	5.13×10^5	189

*$Q_g = 173.9\,kJ\,mol^{-1}$ [49].
†T = hold time only at tension.

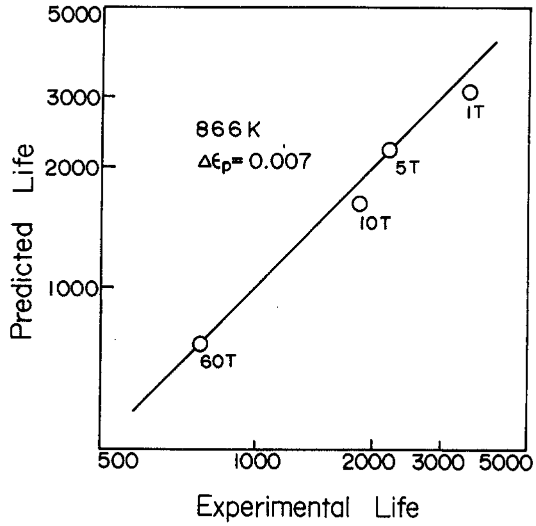

Figure 2 Predicted life against experimental life for 304 stainless steel [39].

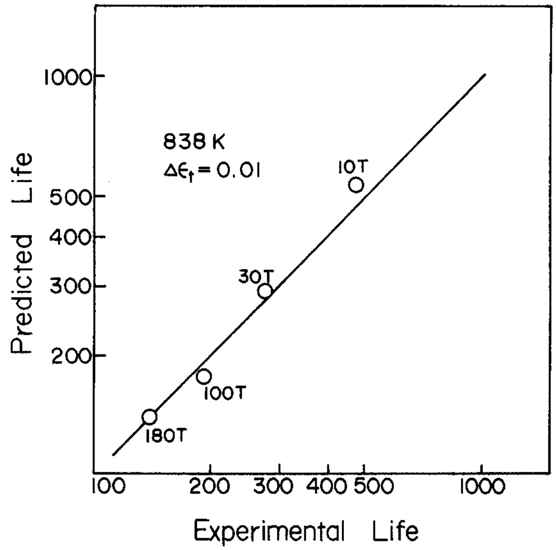

Figure 4 Predicted life against experimental life for CrMoV steel [41]

cavitation during fatigue is believed to be physically feasible.

An important advantage of this model is its ability to handle creep–fatigue behaviour over a wide range of temperature compared to phenomenological approaches.

4. Conclusion

1. Failure by creep–fatigue is controlled by round-type cavitation damage rather than by a process of crack initiation and propagation.

2. Cavities are formed in every cycle and the number of cavities per cycle is proportional to the plastic strain range. This is represented by $n = P\Delta\varepsilon_p N$.

3. Cavity growth is controlled by vacancy diffusion through grain boundaries.

4. The number of cycles to failure is given by

$$N = C\left(\frac{\int_0^t \sigma(t)\,dt \exp(-Q_g/RT)\Delta\varepsilon_p^{3/2}}{T}\right)^{-2/5}$$

5. The proposed model based on round-type creep cavitation is compared with creep–fatigue data reported by other investigators; there is good agreement between them.

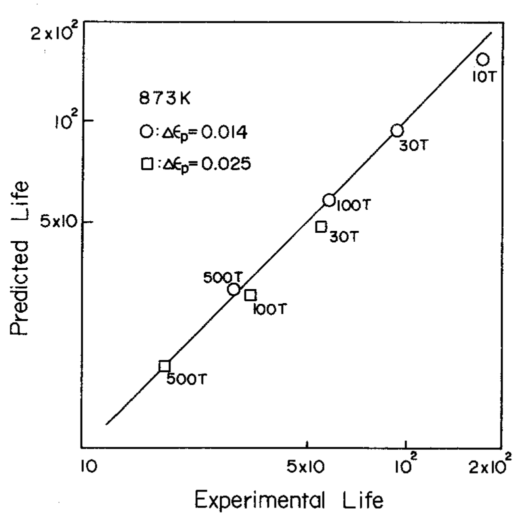

Figure 3 Predicted life against experimental life for 316 stainless steel [40].

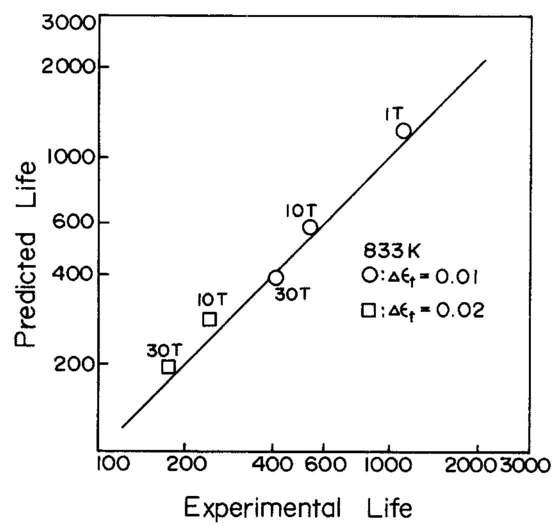

Figure 5 Predicted life against experimental life for 13CrMo44 steel [42].

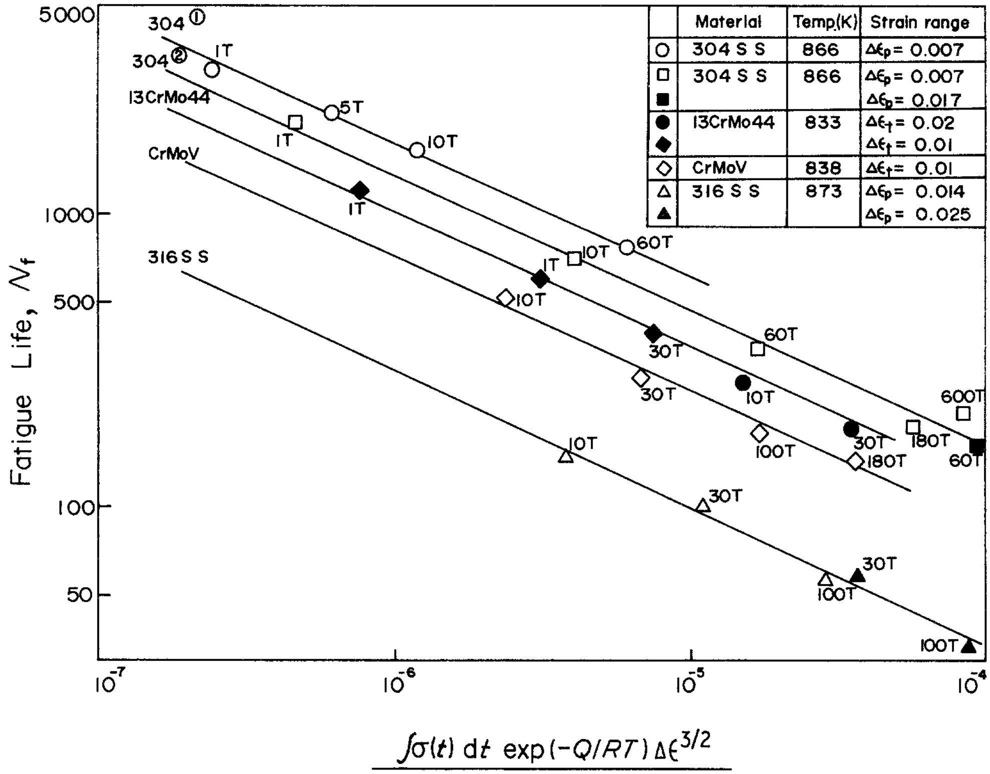

Figure 6 Fatigue life against $\int_0^t \sigma(t)\,dt$ normalized by temperature and strain range for various materials.

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